ISSN 0868–5886 NAUCHNOE PRIBOROSTROENIE, 2016, Vol. 26, No. 3, pp. 83–94

MATHEMATICAL METHODS AND MODELLING IN INSTRUMENT MAKING

UDC 519.2

© A. S. Ilyin (ORCID: 0000-0001-8426-5580)

BASES OF DYNAMIC CORRECTION OF SENSITIVITY COEFFICIENTS OF GROUP OF THE SAME SENSORS (ON THE EXAMPLE OF COSINE DISTRIBUTION LAW)

It is supposed that at an operational phase there is an opportunity to perform measurement of an isotropic background. Carrying out sorting of the measured values in group of the same sensors, in case of similarity of their sensitivity we receive for each sensor a uniform picture of distribution on positions of the sorted list. In case of sensitivity drift of one sensor there is an imbalance: the measured values from this sensor appear in one half more often than in another. Formulas and results of calculations establishing dependence of an imbalance on the size of drift of one sensor as a basis for dynamic correction of coefficients of sensitivity of the same sensors are presented. Similarity of result for the even number of sensors and bigger odd is proved. As an example cosine distribution of the measured random variable is considered. A choice of parameters of this distribution by some criteria of likeness to normal distribution is made.

Кл. сл.: intelligent sensors, sensitivity drift, adjustment, accuracy, survivability, self-restoration

INTRODUCTION

Sensors of various types are the most important part of modern robotic and human-machine systems. At the same time high requirements to survivability and accuracy of measurements in combination with impossibility or difficulty of involvement of the person in execution of rescue and recovery operations in extremal operating conditions generate actual need of intellectualization of sensors. In [1] it is specified that, unlike the adaptive sensor possessing properties of automatic control of the parameters depending on the current conditions, the intelligent sensor in addition has to carry out:

– automatic correction of the error which has appeared as a result of influence of any sizes and/or aging of components;

– self-restoration at emergence of single defect in the sensor;

– self-training.

It is known that the wildlife provides us examples of plurality of receptors as a part of an eye and an ear. Sensors of technical systems are in many respects similar to organs of vision and hearing. This similarity can be shown differently, but within a subject of this article is pertinent to pay attention to the following examples of application of a set of the same sensors:

– the acoustic system containing a number of microphones around a tower as a part of the fighting robot "Scorpion" [2] provides an opportunity instantly and very precisely to define the direction on a source of a sound (shot) respectively to direct a gun trunk;

– the dose rate measuring unit (DRMU) [3–5] containing a number of sensors (Geiger counters) around the cylinder from a wolfram alloy also provides to the user a direction angle on a radiation source.

Characteristic of systems of such look is need of similarity of sensitivity of sensors. Meanwhile parameters of various made counters of radiation registration are presented in [6]. In the column "Sensitivity" it is specified, for example: 60–75 imp/μR for the Gamma-7 counter; 285–385 imp/μR for the Gamma-8 counter; 31–39 imp/μR for the Gamma-10 counter. We see that each model allows initial dispersion about 25 %. In this regard tuning of parameters of the software (sensitivity coefficients) in devices first of all at a fabrication stage, as well as periodically at an operational phase is performed at scheduled works, with use of a standard source.

But if we speak about relevance of intellectualization of touch systems, then we will set the task of dynamic correction of sensitivity coefficients, i.e. directly during working functioning at an operational phase, without standard source.

Such opportunity exists if it is allowed to consider that the background which is saved up for the long time in intervals between situations of existence of sources possesses property of isotropy.

Relevance of improvement of means of radiation control is specified, for example, in [7]. It is supposed to use actively mobile land, air and underwater robotic means. Means of radiation control can be also stationary, established in remote, dangerous and hard-to-reach spots. In all options the intellectual measuring means possessing property of self-restoration are more preferable.

PROBLEM DEFINITION

We consider that we know dependence of a population mean of the measured value on sensor settings. Respectively we have an opportunity to manipulate shifts. In particular, as a part of the software of DRMU for each sensor the sensitivity coefficient which nominal rate is equal to 1 is provided.

The purpose of article is receiving formulas on the basis of which an opportunity to calculate the size of drift of the sensor will be provided that then to make recalculation of its settings (sensitivity coefficient). For this purpose formation of source data as follows has to be provided in the software of a unit.

Checking lack of a radiation source (lack of excessive unevenness of results of the measurements arriving from sensors), we carry out accumulation of a background during the current interval of time (step). More precisely, it is more correct to consider that the level of a background can change therefore each step of accumulation of a background has to come to the end when the average (or median) value of result of accumulation reaches the demanded value. On each step we carry out sorting of the values received from all sensors. We form the array of counters $B_{i,j}$ - quantity of cases of hit of i-th sensor in j-th position of the sorted list of values. We make counting of indexes from zero.

Let M — quantity of steps; N — quantity of counters.

It is obvious that for every i equality works:

$$M = \sum_{j=0}^{N-1} B_{i,j} .$$

For each sensor we calculate the rated sums on a half of counters:

$$Q_i = \frac{1}{M} \sum_{j=0}^{\frac{N}{2}-1} B_{i,j} \qquad \text{at even quantity of } N,$$

$$Q_i = \frac{1}{M} \left(\frac{1}{2} B_{i,\frac{N-1}{2}} + \sum_{j=0}^{\frac{N-1}{2}-1} B_{i,j} \right) \qquad \text{at odd quantity of } N.$$

We will call each such sum of Q_i an imbalance. In the absence of drift (in an initial state) the imbalance is equal to 1/2, and at increase in drift up to the full width of statistical dispersion the imbalance becomes equal to 1.

It is clear, that the drifting sensor gets the greatest imbalance, and other sensors get smaller (evenly distributed between them) an imbalance of an opposite orientation.

BASIC FORMULAS

All sensors are subject to drift, but in the range of time between sessions of recalculation of sensitivity coefficients excessively increased drift is most probable only at one sensor. Therefore within this article in the used mathematical model it is supposed that only one sensor is subject to drift.

Let $p(a,x)$ — probability density function (PDF) of the measured size x in the field of values which width is characterized by parameter a. Without restriction of a community for convenience of calculations it is possible to consider that average value is equal to zero.

Let L — drift size towards understating. It means that PDF takes form $p(a, x + L)$.

We will designate also cumulative distribution function (CDF):

$$P\left(a,X\right) = \int_{-\infty}^{X} p\left(a,x\right) \mathrm{d}x. \qquad (1)$$

We will write down probability of hit of the drifting sensor in j-th position of sorted list (we count indexes from zero):

$$q\left(a,L,N,j\right)=\frac{\left(N-1\right)!}{j!\left(N-1-j\right)!}\times$$
$$\times\int_{-\infty}^{\infty}P\left(a,x\right)^{j}\left(1-P\left(a,x\right)\right)^{N-1-j}p\left(a,x+L\right)\mathrm{d}x. \qquad (2)$$

Here in integrand expression:

– the first multiplier corresponds to j sensors at which the measured value has appeared less, than x;

– the second multiplier corresponds to $(N-1-j)$ sensors at which the measured value has appeared more, than x;

– the third multiplier defines probability that the drifting sensor gives value in the range from x to $x+\mathrm{d}x$.

The combinatory coefficient considers shifts of all sensors, except drifting.

It is pertinent to notice that the formula (2) defines probability that value from the drifting sensor is j-th order statistic. Unlike the formulas which are available, for example, in [8] or [9], written down for identical sensors, for variant with the drifting sensor every moment of j-th order statistict can be written down in the form of the sum of three sizes (as in article [10]); (2) is one of these three sizes for the 0-order moment.

We will designate a middle index:

at even N: $$n=\frac{N}{2}-1, \qquad (3)$$

at odd N: $$n=\frac{N-1}{2}. \qquad (4)$$

We will designate a middle multiplier:

at even N: $$\delta\left(j\right)=1, \qquad (5)$$

at odd N: $$\delta\left(j\right)=1 \text{ при } j\neq n,$$
$$\delta\left(n\right)=1/2. \qquad (6)$$

The goal of article is formalized as computation of the amount of probabilities of hit on a half of the sorted list:

$$Q\left(a,L,N\right)=\sum_{j=0}^{n}\delta\left(j\right)q\left(a,L,N,j\right). \qquad (7)$$

We will consider a particular case L=0. There is an opportunity to integrate a formula (2) "in parts" as follows:

$$q\left(a,0,N,j\right)=\frac{\left(N-1\right)!}{j!\left(N-1-j\right)!}\times$$
$$\times\int_{-\infty}^{\infty}P\left(a,x\right)^{j}\left(1-P\left(a,x\right)\right)^{N-1-j}p\left(a,x\right)\mathrm{d}x=$$
$$=\frac{-\left(N-1\right)!}{j!\left(N-j\right)!}\left[P\left(a,x\right)^{j}\left(1-P\left(a,x\right)\right)^{N-j}\right]_{-\infty}^{\infty}+\ldots+$$
$$+\frac{\left(N-1\right)!}{\left(j-1\right)!\left(N-j\right)!}\int_{-\infty}^{\infty}P\left(a,x\right)^{j-1}\left(1-P\left(a,x\right)\right)^{N-j}p\left(a,x\right)\mathrm{d}x.$$

The integral which is expression $q(a,0,N,j-1)$ turned out.

Therefore it is convenient to execute summing on a formula (7) in decreasing order of an index j. The calculated amount takes a form:

$$Q\left(a,0,N\right)=\sum_{j=0}^{n}\left\{\left(n-j+\delta\left(n\right)\right)\frac{-\left(N-1\right)!}{j!\left(N-j\right)!}\times\right.$$
$$\left.\times\left[P\left(a,x\right)^{j}\left(1-P\left(a,x\right)\right)^{N-j}\right]_{-\infty}^{\infty}\right\}.$$

The appeared multiplier $\left(n-j+\delta\left(n\right)\right)$ is a consequence of combining of similar members. Considering that $P\left(a,-\infty\right)=0$ and $P\left(a,\infty\right)=1$, it is easy to see that in this amount there is only one nonzero member having the j=0 index and lower bound of an interval of integration. Taking into account (3)–(6), as one would expect, we receive identity:

$$Q\left(a,0,N\right)=\frac{1}{2}. \qquad (8)$$

For the considered case L=0 it was unessential whether the area of integration of the infinite is or it is restricted to a segment.

We will read further that the range of accidental dispersion of the measured value is limited by an interval $\left[-a,a\right]$. At the same time it is necessary to rewrite formulas (2) and (7) a little differently.

We will note that if the drifting sensor gives value less, than $-a$, then it certainly gets to a zero line item. Such event arises with probability $\left[-a,a-L\right]$ is an integration interval for a formula (2). Therefore:

$$q(a,L,N,j)=\frac{\left(N-1\right)!}{j!\left(N-1-j\right)!}\times$$
$$\times\int_{-a}^{a-L}P\left(a,x\right)^{j}\left(1-P\left(a,x\right)\right)^{N-1-j}p\left(a,x+L\right)\mathrm{d}x, \qquad (9)$$

$$Q\left(a,L,N\right)=P(a,L-a)+\sum_{j=0}^{n}\delta\left(j\right)q\left(a,L,N,j\right). \qquad (10)$$

For a formula (9) we will execute integration "in parts":

$$q(a,L,N,j)=\frac{(N-1)!}{j!(N-1-j)!}\times$$
$$\times\Big[P(a,x)^j\big(1-P(a,x)\big)^{N-1-j}P(a,x+L)\Big]_{-a}^{a-L}-$$
$$-\frac{(N-1)!}{(j-1)!(N-1-j)!}\int_{-a}^{a-L}P(a,x)^{j-1}\big(1-P(a,x)\big)^{N-1-j}\times$$
$$\times p(a,x)P(a,x+L)\mathrm{d}x+$$
$$+\frac{(N-1)!}{j!(N-2-j)!}\int_{-a}^{a-L}P(a,x)^{j}\big(1-P(a,x)\big)^{N-2-j}\times$$
$$\times p(a,x)P(a,x+L)\mathrm{d}x. \qquad (11)$$

In case of j=0 in this formula the first integral is absent, and in case of j=1 completely (except a sign) matches the second integral if to consider it in case of j=0. Similarly and in case of the following values of an index j. Therefore when summing on a formula (10) there is a mutual destruction of all integrals, except one or two integrals (depending on parity of N) with middle indexes.

In case of even N amount of integrals turns into one integral:

$$I(a,L,N)=$$
$$=\frac{(N-1)!}{\left(\frac{N}{2}-1\right)!\left(\frac{N}{2}-1\right)!}\times$$
$$\times\int_{-a}^{a-L}P(a,x)^{\frac{N}{2}-1}\big(1-P(a,x)\big)^{\frac{N}{2}-1}p(a,x)P(a,x+L)\mathrm{d}x. \qquad (12)$$

In case of odd N amount of integrals turns into two integrals. At the same time, considering (4) and (6), we find an opportunity to execute summing and to turn them into one integral:

$$I(a,L,N)=$$
$$=\frac{(N-1)!}{2(n-1)!(N-1-n)!}\int_{-a}^{a-L}P(a,x)^{n-1}\big(1-P(a,x)\big)^{N-1-n}p(a,x)P(a,x+L)\mathrm{d}x+$$
$$+\frac{(N-1)!}{2n!(N-2-n)!}\int_{-a}^{a-L}P(a,x)^{n}\big(1-P(a,x)\big)^{N-2-n}p(a,x)P(a,x+L)\mathrm{d}x=$$
$$=\frac{(N-1)!}{2\left(\frac{N-1}{2}\right)!\left(\frac{N-1}{2}-1\right)!}\int_{-a}^{a-L}P(a,x)^{\frac{N-1}{2}-1}\big(1-P(a,x)\big)^{\frac{N-1}{2}-1}p(a,x)P(a,x+L)\mathrm{d}x. \qquad (13)$$

In a formula (11) we execute substitution of limits of integration:

$$\sum_{j=0}^{n}\left\{\delta(j)\frac{(N-1)!}{j!(N-1-j)!}\times\right.$$
$$\left.\times\Big[P(a,x)^j\big(1-P(a,x)\big)^{N-1-j}P(a,x+L)\Big]_{-a}^{a-L}\right\}=$$
$$=D(a,L,N)-P(a,L-a).$$

Here the amount is designated:

$$D(a,L,N)=\sum_{j=0}^{n}\left[\delta(j)\frac{(N-1)!}{j!(N-1-j)!}\times\right.$$
$$\left.\times P(a,a-L)^j\big(1-P(a,a-L)\big)^{N-1-j}\right].$$

The formula (10) takes a form:

$$Q(a,L,N)=D(a,L,N)+I(a,L,N). \qquad (14)$$

It is easy to be convinced that, in particular:

$$D(a,0,N)=0;\quad D(a,a,N)=\frac{1}{2};$$
$$D(a,2a,N)=1;\quad I(a,2a,N)=0.$$

Considering (8) and (14), it is possible to write one more identity: $I(a,0,N)=\frac{1}{2}$.

These identities can be the useful in case of programming validation.

COSINE DISTRIBUTION

In practice usually we deal with normal distribution of the accidental values received from sensors. It is known that Poisson distribution (data from counters of registration of radioactive radiation.) also is close to normal distribution. However here we will consider cosine distribution of accidental values and we will find its parameters in case of which it has the maximum likeness to normal distribution. Unlike exponential Laplace function, cosine function allows analytical computation of integral (12). More precisely, further we will see that the turning-out result has an appearance of the amount which computation is not the most complex challenge from the point of view of programming.

We will define PDF of cosine distribution in the following look:

$$p(a,\lambda,x)=\frac{1+\lambda\cos\left(\pi\dfrac{x}{a}\right)}{2a}. \tag{15}$$

The amplitude parameter intended for regulation of sharpness of a distribution function has the range:

$$0\le\lambda\le 1. \tag{16}$$

CDF of cosine distribution:

$$P(a,\lambda,x)=\frac{\dfrac{x}{a}+1+\dfrac{\lambda}{\pi}\sin\left(\pi\dfrac{x}{a}\right)}{2} \tag{17}$$

Applying the integration formulas known, for example, from [11, p. 89] or [12], it is easy to receive dispersion (moment of the second order) of cosine distribution

$$2\int_0^a\frac{1+\lambda\cos\left(\pi\dfrac{x}{a}\right)}{2a}x^2\mathrm{d}x=\left(\frac{1}{3}-\frac{2\lambda}{\pi^2}\right)a^2$$

and moment of the fourth order

$$2\int_0^a\frac{1+\lambda\cos\left(\pi\dfrac{x}{a}\right)}{2a}x^4\mathrm{d}x=a^4\left(\frac{1}{5}-\lambda\frac{4\pi^2-24}{\pi^4}\right).$$

According to the determination specified, for example, in [13, p. 18], we receive excess coefficient:

$$E(\lambda)=\frac{\dfrac{\pi^4}{5}-4\lambda\left(\pi^2-6\right)}{\left(\dfrac{\pi^2}{3}-2\lambda\right)^2}-3. \tag{18}$$

For comparing with normal distribution further we will need the known formulas which here we will write in the following designations:

$$g(\sigma,x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right), \tag{19}$$

$$G(\sigma,x)=\frac{1}{2}\left(1+\operatorname{erf}\left(\frac{x}{\sigma\sqrt{2}}\right)\right), \tag{20}$$

$$\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x\exp\left(-t^2\right)\mathrm{d}t.$$

LIKENESS OF COSINE DISTRIBUTION TO NORMAL DISTRIBUTION BY CRITERIA OF DISPERSION AND EXCESS

The received function of excess coefficient (18) has peak point.

To find its coordinates, it is necessary to calculate a derivative by the parameter λ and to equate to zero. It is easy to receive the decision:

$$\lambda_{E_{\max}}=\frac{\pi^2\left(15-\pi^2\right)}{15\left(\pi^2-6\right)}=0.872354, \tag{21}$$

$$E_{\max}=\frac{15\left(\pi^2-6\right)^2}{\pi^2\left(7\pi^2-60\right)}-3=-0.495661. \tag{22}$$

The received value of amplitude coefficient (21) appeared in boundaries of the range (16) intended for regulation of sharpness of a PDF. The appropriate maximum value of excess coefficient (22) most close approaches the zero level. It means that sharpness of cosine distribution in this case is closest to sharpness of normal distribution.

In addition to the requirement of likeness of excess coefficients it is offered to equate dispersions. It is easy to receive the following ratio:

$$a=\sigma\sqrt{\frac{15\left(\pi^2-6\right)}{7\pi^2-60}}=2.527337\sigma. \tag{23}$$

The function graph (19) with parameter $\sigma=1$ and a function graph (15) with parameters (21) and (23) are provided in a fig. 1. We see that this criterion of likeness based on closeness of integral statistical characteristics tries to increase the cut-down "tail" due to reduction of peak.

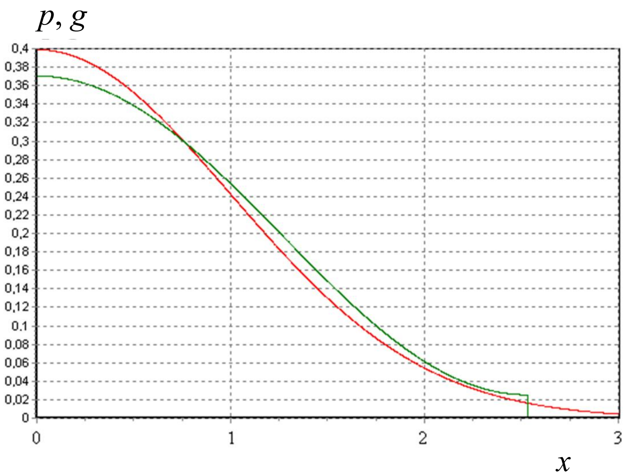

Fig. 1. Comparing of cosine distribution p with normal distribution g by criterion of closeness of dispersion and an excess

CRITERION OF UNIFORM LIKENESS OF COSINE DISTRIBUTION TO NORMAL DISTRIBUTION

Formulation of criterion of uniform likeness

We will consider also criterion of uniform likeness of functions, namely we will execute minimization of integral from a square of their difference:

$$F(a,\lambda,\sigma)=\int_0^a \left(p(a,\lambda,x)-g(\sigma,x)\right)^2 \mathrm{d}x+$$
$$+\int_a^\infty \left(g(\sigma,x)\right)^2 \mathrm{d}x.$$

These integrals allow a regrouping in which only one of two integrals plays a role for the task of minimization:

$$F(a,\lambda,\sigma)=\int_0^a p(a,\lambda,x)\left(p(a,\lambda,x)-2g(\sigma,x)\right)\mathrm{d}x+$$
$$+\int_0^\infty \left(g(\sigma,x)\right)^2 \mathrm{d}x.$$

We will work out system of two equations concerning parameters a and λ:

$$\frac{\mathrm{d}F(a,\lambda,\sigma)}{\mathrm{d}\lambda}=0, \tag{24}$$

$$\frac{\mathrm{d}F(a,\lambda,\sigma)}{\mathrm{d}a}=0. \tag{25}$$

Conversion of the equation (24)

By the famous rules of computation derivative of integral we receive:

$$\int_0^a \left(\frac{1+\lambda\cos\left(\pi\frac{x}{a}\right)}{2a}-\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)\times$$
$$\times\frac{\cos\left(\pi\frac{x}{a}\right)}{2a}\mathrm{d}x=0. \tag{26}$$

We will transform to a type of two separate integrals:

$$\lambda\int_0^a \cos^2\left(\pi\frac{x}{a}\right)\mathrm{d}x=\frac{a\sqrt{2}}{\sigma\sqrt{\pi}}\int_0^a \exp\left(-\frac{x^2}{2\sigma^2}\right)\cos\left(\pi\frac{x}{a}\right)\mathrm{d}x.$$

The integral in the left part can be taken on the known formula [11, p. 90], a/2 turns out.

We consider integral in the right part as the function calculated by numerical integration.

We will designate:

$$u=\frac{a}{\sigma\sqrt{2}},$$
$$H(z)=\int_0^1 \exp\left(-zt^2\right)\cos(\pi t)\mathrm{d}t. \tag{27}$$

The equation (24) takes a form:

$$\lambda=\frac{4u}{\sqrt{\pi}}H\left(u^2\right) \tag{28}$$

Conversion of the equation (25)

$$\frac{\mathrm{d}F(a,\lambda,\sigma)}{\mathrm{d}a}=p(a,\lambda,a)\left(p(a,\lambda,a)-2g(\sigma,a)\right)+$$
$$+2\int_0^a \frac{\mathrm{d}p(a,\lambda,x)}{\mathrm{d}a}\left(p(a,\lambda,x)-g(\sigma,x)\right)\mathrm{d}x=0;$$

$$2\int_0^a \left(\frac{1+\lambda\cos\left(\pi\frac{x}{a}\right)}{2a}-\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)\times$$
$$\times\left(\frac{\lambda\pi\frac{x}{a^2}\sin\left(\pi\frac{x}{a}\right)}{2a}-\frac{1+\lambda\cos\left(\pi\frac{x}{a}\right)}{2a^2}\right)\mathrm{d}x+$$

$$+\left(\frac{1+\lambda\cos\left(\pi\frac{a}{a}\right)}{2a}-\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right)\right)^2-$$

$$-\left(\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right)\right)^2=0.$$

The second equation contains the left part of the first equation (26) which is equal to zero therefore there is a possibility of simplification:

$$\int_0^a\left(\frac{1+\lambda\cos\left(\pi\frac{x}{a}\right)}{2a}-\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)\times$$

$$\times\left(\frac{\lambda\pi\frac{x}{a^2}\sin\left(\pi\frac{x}{a}\right)}{a}-\frac{1}{a^2}\right)\mathrm{d}x+$$

$$+\frac{(1-\lambda)^2}{4a^2}-\frac{(1-\lambda)}{a\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right)=0\,.$$

We will transform to a type of three separate integrals:

$$\int_0^a\left(\frac{1+\lambda\cos\left(\pi\frac{x}{a}\right)}{2a}-\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)\times$$

$$\times\lambda\pi\frac{x}{a}\sin\left(\pi\frac{x}{a}\right)\mathrm{d}x=$$

$$=\int_0^a\frac{1+\lambda\cos\left(\pi\frac{x}{a}\right)}{2a}\mathrm{d}x-\int_0^a\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\mathrm{d}x-$$

$$-\frac{(1-\lambda)^2}{4}+\frac{(1-\lambda)a}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right).$$

Here "tail" of normal distribution which quits abroad cosine distribution is viewed. Therefore we execute further simplification:

$$\int_0^a\left(\frac{1+\lambda\cos\left(\pi\frac{x}{a}\right)}{2a}-\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)\times$$

$$\times\lambda\pi\frac{x}{a}\sin\left(\pi\frac{x}{a}\right)\mathrm{d}x=$$

$$=1-G(\sigma,a)-\frac{(1-\lambda)^2}{4}+\frac{(1-\lambda)a}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right).$$

We will take the remained integral "in parts":

$$-\lambda\left[\left(\frac{1+\lambda\cos\left(\pi\frac{x}{a}\right)}{2a}-\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)x\cos\left(\pi\frac{x}{a}\right)\right]_0^a+$$

$$+\lambda\int_0^a\left(\frac{1+\lambda\cos\left(\pi\frac{x}{a}\right)}{2a}-\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)\cos\left(\pi\frac{x}{a}\right)\mathrm{d}x+$$

$$+\lambda\int_0^a\left(-\frac{\lambda\pi\sin\left(\pi\frac{x}{a}\right)}{2a^2}+\frac{x}{\sigma^3\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)x\cos\left(\pi\frac{x}{a}\right)\mathrm{d}x=$$

$$=1-G(\sigma,a)-\frac{(1-\lambda)^2}{4}+\frac{(1-\lambda)a}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right).$$

The first of two remained integrals is multiple the left part of the first equation (26) therefore it is equal to zero. Expression in square brackets it is concretized and portable in the right member of equation:

$$\lambda\int_0^a\left(-\frac{\lambda\pi\sin\left(\pi\frac{x}{a}\right)}{2a^2}+\frac{x}{\sigma^3\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)x\cos\left(\pi\frac{x}{a}\right)\mathrm{d}x=$$
$$=1-G(\sigma,a)-\frac{(1-\lambda)^2}{4}+\frac{(1-\lambda)a}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right)-\lambda a\left(\frac{1-\lambda}{2a}-\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right)\right).$$

In the right part we add similar members:

$$\lambda\int_0^a\left(-\frac{\lambda\pi\sin\left(\pi\frac{x}{a}\right)}{2a^2}+\frac{x}{\sigma^3\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)\times$$
$$\times x\cos\left(\pi\frac{x}{a}\right)\mathrm{d}x=$$
$$=1-G(\sigma,a)-\frac{(1-\lambda)^2}{4}+\frac{a}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right)-\frac{\lambda(1-\lambda)}{2}.$$

In the left part we break integral into two integrals, we apply a sine of a double angle; in the right part we add similar members:

$$\frac{\lambda}{\sigma^3\sqrt{2\pi}}\int_0^a\exp\left(-\frac{x^2}{2\sigma^2}\right)\cos\left(\pi\frac{x}{a}\right)x^2\mathrm{d}x-$$
$$-\frac{\lambda^2\pi}{4a^2}\int_0^a\sin\left(2\pi\frac{x}{a}\right)x\mathrm{d}x=$$
$$=\frac{3}{4}+\frac{\lambda^2}{4}-G(\sigma,a)+\frac{a}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right).$$

On the formula known from [11, p. 84], we receive:

$$\frac{\lambda^2\pi}{4a^2}\int_0^a\sin\left(2\pi\frac{x}{a}\right)x\mathrm{d}x=-\frac{\lambda^2}{8}.$$

We transfer the calculated integral to the right part and we add similar members:

$$\frac{\lambda a^3}{\sigma^3\sqrt{2\pi}}\int_0^1\exp\left(-\frac{a^2}{2\sigma^2}t^2\right)\cos(\pi t)t^2\mathrm{d}t=$$
$$=\frac{1}{4}+\frac{\lambda^2}{8}-\frac{1}{2}\mathrm{erf}\left(\frac{a}{\sigma\sqrt{2}}\right)+\frac{a}{\sigma\sqrt{2\pi}}\exp\left(-\frac{a^2}{2\sigma^2}\right).$$

We will use designation (27), and also we will designate the function calculated by numerical integration:

$$T(z)=\int_0^1\exp\left(-zt^2\right)\cos(\pi t)t^2\mathrm{d}t.$$

The equation (25) takes a form of a quadratic equation:

$$\lambda^2-\lambda\frac{16u^3}{\sqrt{\pi}}T\left(u^2\right)+$$
$$+2-4\,\mathrm{erf}(u)+\frac{8u}{\sqrt{\pi}}\exp\left(-u^2\right)=0. \quad (29)$$

Decision of system of two equations

By programming the following decision of system of two equations (28) and (29) is received:

$$\begin{aligned}\lambda&=0.857,\\ u&=1.671;\end{aligned} \quad (30)$$

$$a=2.363\sigma. \quad (31)$$

The function graph of normal distribution (19) with parameter $\sigma=1$ and a function graph of cosine distribution (15) with parameters (30) and (31) are provided in a fig. 2. Apparently, the considered criterion provides the very close likeness.

CALCULATION FORMULAS OF IMBALANCE FUNCTION FOR COSINE DISTRIBUTION

The integral (12) for (14) after substitution (15) and (17) takes a form:

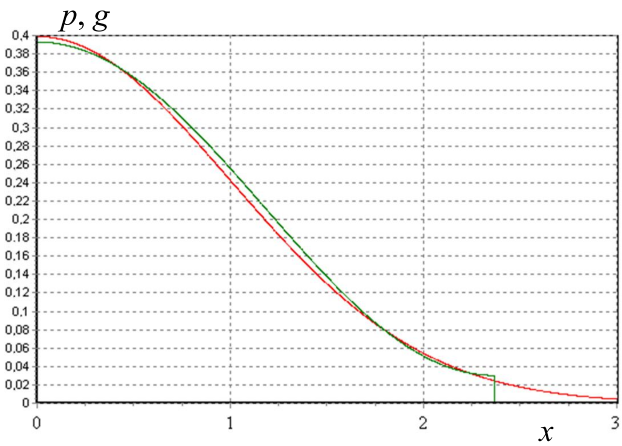

Fig. 2. Comparing of cosine distribution p with normal distribution g by criterion of uniform likeness

$$I(a,L,N)=\frac{(N-1)!}{\left(\frac{N}{2}-1\right)!\left(\frac{N}{2}-1\right)!}\times$$

$$\times\int_{-a}^{a-L}\left[\left(\frac{a+x+\lambda\frac{a}{\pi}\sin\left(\frac{\pi x}{a}\right)}{2a}\right)^{n}\left(\frac{a-x-\lambda\frac{a}{\pi}\sin\left(\frac{\pi x}{a}\right)}{2a}\right)^{n}\times\right.$$

$$\left.\times\frac{1+\lambda\cos\left(\frac{\pi x}{a}\right)}{2a}\,\frac{a+x+L+\lambda\frac{a}{\pi}\sin\left(\frac{\pi}{a}(x+L)\right)}{2a}\right]\mathrm{d}x.$$

We will designate $z=L/a$; $t=x/a$; we will also rewrite prior expression in these designations (we will note that, as well as the excess coefficient, imbalance function depends only on the form of PDF, but not on its width):

$$I(z,N)=\frac{(N-1)!}{n!^{2}\,2^{N}}\int_{-1}^{1-z}\left[\left(1-\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2}\right)^{n}\times\right.$$

$$\left.\times\left(1+\lambda\cos(\pi t)\right)\left(1+t+z+\frac{\lambda}{\pi}\sin\left(\pi(t+z)\right)\right)\right]\mathrm{d}t.$$

We will execute expansion of a sine of the amount; and we will create the member matching the member wich is available in the base of exponentiation at integrand expression:

$$\left(1+t+z+\frac{\lambda}{\pi}\sin\left(\pi(t+z)\right)\right)=$$
$$=1+z+t\left(1-\cos(\pi z)\right)+$$
$$+\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)\cos(\pi z)+\frac{\lambda}{\pi}\cos(\pi t)\sin(\pi z).$$

It allows us to write the required integral in the following look:

$$I(z,N)=\frac{(N-1)!}{n!^{2}\,2^{N}}\Big((1+z)U+\left(1-\cos(\pi z)\right)V+$$
$$+\cos(\pi z)R+\frac{\lambda}{\pi}\sin(\pi z)S\Big).$$

Here are designated:

$$U=\int_{-1}^{1-z}\left(1-\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2}\right)^{n}\left(1+\lambda\cos(\pi t)\right)\mathrm{d}t,$$

$$V=\int_{-1}^{1-z}\left(1-\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2}\right)^{n}\left(1+\lambda\cos(\pi t)\right)t\,\mathrm{d}t,$$

$$R=\int_{-1}^{1-z}\left(1-\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2}\right)^{n}\times$$
$$\times\left(1+\lambda\cos(\pi t)\right)\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)\mathrm{d}t,$$

$$S=\int_{-1}^{1-z}\left(1-\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2}\right)^{n}\times$$
$$\times\left(1+\lambda\cos(\pi t)\right)\cos(\pi t)\,\mathrm{d}t.$$

From these four integrals two integrals it is possible to take easily:

$$U=\sum_{k=0}^{n}\frac{(-1)^{k}}{2k+1}C_{n}^{k}\left[\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2k+1}\right]_{-1}^{1-z},$$

$$R=\sum_{k=0}^{n}\frac{(-1)^{k}}{2k+2}C_{n}^{k}\left[\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2k+2}\right]_{-1}^{1-z}.$$

We apply a method "in parts" to two other integrals:

$$V=\sum_{k=0}^{n}\frac{(-1)^{k}}{2k+1}C_{n}^{k}\left[\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2k+1}t\right]_{-1}^{1-z}-$$

$$-\sum_{k=0}^{n}\frac{(-1)^k}{2k+1}C_n^k\int_{-1}^{1-z}\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2k+1}\mathrm{d}t\,;$$

$$S=\sum_{k=0}^{n}\frac{(-1)^k}{2k+1}C_n^k\left[\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2k+1}\cos(\pi t)\right]_{-1}^{1-z}+$$

$$+\pi\sum_{k=0}^{n}\frac{(-1)^k}{2k+1}C_n^k\int_{-1}^{1-z}\left(t+\frac{\lambda}{\pi}\sin(\pi t)\right)^{2k+1}\sin(\pi t)\mathrm{d}t.$$

Further there is opportunity in integrand expressions also to execute binomial expansion, and then to apply formulas from [14, pp. 197, 198]:

$$\int x^n\sin^{2m}ax\,\mathrm{d}x=\frac{(2m)!}{m!m!}\frac{x^{n+1}}{2^{2m}(n+1)}+$$

$$+\frac{(-1)^m}{2^{2m-1}}\sum_{k=0}^{m-1}(-1)^k\frac{(2m)!}{k!(2m-k)!}\int x^n\cos((2m-2k)ax)\mathrm{d}x;$$

$$\int x^n\sin^{2m+1}ax\,\mathrm{d}x=$$

$$=\frac{(-1)^m}{2^{2m}}\sum_{k=0}^{m}\frac{(-1)^k(2m+1)!}{k!(2m+1-k)!}\int x^n\sin((2m-2k+1)ax)\mathrm{d}x;$$

$$\int x^n\sin(ax)\mathrm{d}x=-\sum_{k=0}^{n}\frac{n!}{(n-k)!}\frac{x^{n-k}}{a^{k+1}}\cos\left(ax+\frac{k\pi}{2}\right);$$

$$\int x^n\cos(ax)\mathrm{d}x=\sum_{k=0}^{n}\frac{n!}{(n-k)!}\frac{x^{n-k}}{a^{k+1}}\sin\left(ax+\frac{k\pi}{2}\right).$$

When the variable t has a power "zero", it is required to calculate integral from a power of sine. In this case we apply the known formula from [12], providing reduction of sine power in integrand expression:

$$\int\sin^n(cx)\mathrm{d}x=$$

$$=-\frac{\sin^{n-1}(cx)\cos(cx)}{nc}+\frac{n-1}{n}\int\sin^{n-2}(cx)\mathrm{d}x.$$

The formula is fair in case of $n>0$.

RESULTS OF CALCULATION OF IMBALANCE FUNCTION FOR COSINE DISTRIBUTION

Imbalance function graphs for cosine distribution in case of N=8 are provided in a fig. 3:

– the upper line 1 for $\lambda=1$;

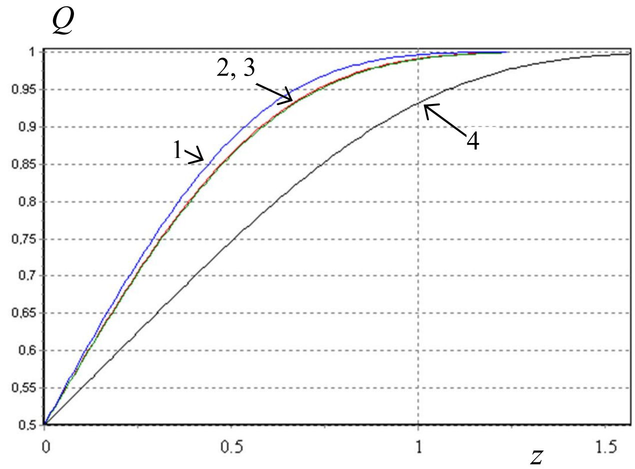

Fig. 3. Lines of an imbalance for cosine distribution.
N = 8; 1 — for λ = 1, 2 — λ = 0.857, 3 — λ = 0.872, 4 — λ = 0

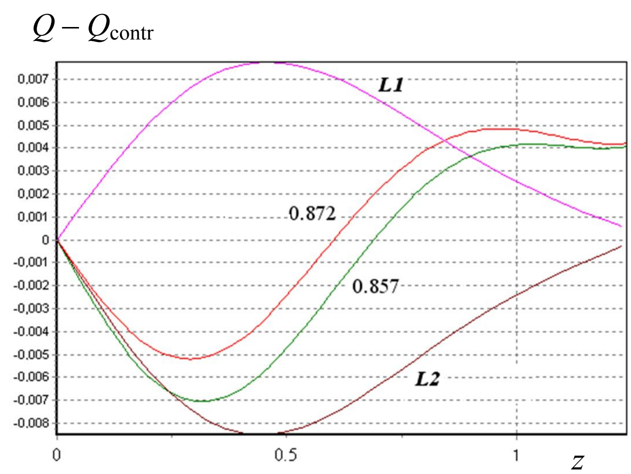

Fig. 4. Lines of an imbalance of rather control line (normal distribution in case of a = 2.44σ).
N = 8; ***L1*** — in case of a ratio (23); ***L2*** — in case of a ratio (31)

– the average line is merge of two close lines 2 and 3 for $\lambda=0.857$ и $\lambda=0.872$;

– the lower line 4 for $\lambda=0$.

It is expedient to analyze the accuracy of approximation of an imbalance for cosine distribution to an imbalance for normal distribution on the basis of numerical integration of a formula (2) with substitution of formulas (19) and (20). At first imbalance function for normal distribution was constructed in two options of horizontal scaling:

– the ***L1*** line based on the ratio (23),

– the ***L2*** line based on the ratio (31).

It turned out that:

L1 passes higher than both lines of an imbalance for cosine distribution;

L2 passes lower than both lines of an imbalance for cosine distribution.

Therefore pertinently to take an average ratio $a = 2.44\sigma$ in case of which the line of an imbalance for normal distribution is built as control line. The built lines of an imbalance rather control line are provided in a fig. 4 in case of $N = 8$.

CONCLUSION

In practice, executing adjustment of sensitivity coefficient, it is expedient to observe some gradualness, but not completely to take the value calculated with use of a mathematical model for which there is a set of assumptions (reliability of monitoring of background isotropy; uniqueness of the sensor subject to drift; compliance to normal distribution). Therefore it is lawful to draw the following conclusions:

1. The provided formulas and results of computation report dependence of an imbalance on value of drift of one sensor as a basis for dynamic adjustment of sensitivity coefficients of the same sensors.

2. The received two lines of an imbalance for cosine distribution are quite acceptable as approaches to the line of an imbalance for normal distribution.

REFERENCES

1. Tajmanov R.E., Sapozhnikova K.V. [Metrological self-checking of sensors]. *Sbornik trudov Vtoroj rossijskoj konferencii s mezhdunarodnym uchastiem "Tehnicheskie i programmnye sredstva sistem upravlenija, kontrolja i izmerenija (teorija, metody, algoritmy, issledovanija i razrabotki)", Sekcija 5: Datchiki v sistemah upravlenija, kontrolja i izmerenija* [Proceedings of the Second Russian conference with the international participation "Technical and software of control systems, control and measurement (the theory, methods, algorithms, researches and development", Section 5: Sensors in control and measurement systems]. Moscow, IPU RAN, October 18–20, 2010. CD-ROM. ISBN 978-5-91450-061-7 (In Russ.).
2. *Dahir Insaat: Combat Robot*. URL: http://fullreels.com/en/video/UZbqsYYapW4/Dahir-Insaat-Combat-Robot-in-Russian (Accessed 22.06.2016) (In Russ.).
3. Vlasenko A.N., Demchenkov V.P., Lapin O.E., Lopota V.A., Nikulenkov K.P., Shelepkov E.A., Judin V.I. *Ustrojstvo dlja izmerenija potokov fotonnogo izluchenija* [The device for measurement of streams of photon radiation]. Patent RF no. 2299450, prioritet 20.05.2007 (In Russ).
4. *Izmeritel' moschnosti dozy i differenzial'nych potokov gamma-izlucheniya IMD-24* [Measuring instrument of power of a dose and differential streams of gamma radiation IMD-24. URL: http://www.rtc.ru/index.php/ru/sredstva-radiatsionnogo-kontrolya/imd-24 (Accessed 22.06.2016) (In Russ.).
5. Arkad'ev V.B., Golubeva O.A., Ilyin A.S., Lapin O.E. [Features of the software of the measuring instrument of power of a dose and differential streams of gamma radiation]. *Rossiyskoe atomnoe soobschestvo. Prezentazii* [The Russian nuclear community. Presentations], 2011, Feb. 28. URL: http://www.atomic-energy.ru/presentations/19074 (Accessed 22.06.2016) (In Russ.).
6. *NPF "Konsensus". Katalog schetchikov registracii izluchenij* [Catalog of counters of registration of radiations]. URL: http://consensus-group.ru/katalog. (Accessed 22.06.2016) (In Russ.).
7. Boyko A.Yu., Vasiliev A.V. [Multi-purpose mobile robotic complex]. *Trudy Mezhdunarodnoj nauchno-tekhnicheskoj konferencii "Ekstremal'naya robototekhnika"* [Proceedings of the International Scientific and Technological Conference "Exteme robotics"], State Scientific Center for Robotics and Technical Cybernetics, October 1–2, 2014, Saint-Petersburg, Russia, pp. 46–49 (In Russ.).
8. David H., Nagaraja H. *Order statistics*. 3rd ed., Wiley, 2003. (Russ. ed.: Dehjvid G. *Porjadkovye statistiki*. Moscow: Glavnaja redakcija fiziko-matematicheskoj literatury Publ., 1979. 336 p.). Doi: 10.1002/0471722162.
9. Gil'bo E.P., Chelpanov I.B. *Obrabotka signalov na osnove uporjadochennogo vybora (mazhoritarnoe i blizkie k nemu preobrazovanija)* [Processing of signals on the basis of the ordered choice (majority and other transformations)]. Moscow, Sovetskoe radio Publ., 1976. 344 p. (In Russ).
10. Ilyin A.S. [Properties of median under drift of one of group of measuring instruments (on the example of uniform distribution)]. *Nauchnoe priborostroenie* [Scientific Instrumentation], 2016, vol. 26, no. 2, pp. 93–100. Doi: 10.18358/np-26-2-i93100.
11. Dwight H.B. *Tables of integrals and other mathematical data*. New York, The Macmillan company Publ., 1961. (Russ. ed.: Dvajt G.B. *Tablicy integralov i drugie matematicheskie formuly*. Moscow, Nauka Publ., 1973. 228 p.).
12. *List of integrals of trigonometric functions*. URL: https://en.wikipedia.org/wiki/List_of_integrals_of_trigonometric_functions (Accessed 22.06.2016).
13. Vadzinsky R.N. *Spravochnik po verojatnostnym raspredelenijam* [Reference book on probabilistic distributions]. St. Petersburg, Nauka Publ., 2001. 296 p. (In Russ.).
14. Gradshtejn I.S., Ryzhik I.M. *Tablicy integralov, summ, rjadov i proizvedenij* [Tables of integrals, sums and serieses] (5th edition). Moscow, Nauka Publ., 1971. 1108 p. (In Russ.).

State Scientific Center for Robotics and Technical Cybernetics, Saint-Petersburg, Russia

Contacts: *Ilyin Anatolij Stepanovich,*
TOLY@RTC.RU

Article received in edition: 21.06.2016