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MATHEMATICAL METHODS AND MODELLING IN INSTRUMENT MAKING

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PROPERTIES OF MEDIAN UNDER DRIFT OF ONE OF GROUP OF MEASURING INSTRUMENTS (ON THE EXAMPLE OF UNIFORM DISTRIBUTION)

The detailed derivation of the formulas for calculation of expected value and dispersion of the median is presented. Thus the uniform law of distribution is considered and it is supposed that data from one of the group of measuring instruments are subject to drift. The number of measuring instruments is odd, therefore as a median we take only one value which appeared in the middle of the sorted list. For the uniform law of distribution it was possible to take integrals and to receive exact analytical formulas, — at some values of the size of drift. The results of calculations, which allow to compare the parameters of the median and arithmetic mean, and also to form the expert opinion on the necessary number of measuring instruments, are presented.

Кл. сл.: median, arithmetic mean, expected value, dispersion, sensitivity drift

INTRODUCTION

Application of a median is necessary everywhere, in particular and when developing devices of radiation control. To current trends of development of intellectual systems of such appointment placement of measuring devices onboard unmanned aerial vehicles (UAVs) is characteristic [1]. At the same time there is an opportunity to choose various types of UAVs, without being limited to an easy class with a loading capacity up to 3 kg. For example, the "Chirok" ("Teal") [2] UAV lifts 275 kg of payload. It means that the dose rate measuring unit (DRMU) [3–5] having the weight of 17 kg developed for land vehicles also can be installed on the UAV. And also it is possible not to be limited to 24 Geiger counters (in the existing developed modification); the way to development of more difficult units is open.

Accumulation of number of sensors is made for increase of measurement accuracy of dose rate and angle of the direction on a radiation source essentially in connection with the casual nature of radioactive radiation. Such need is caused also by limitation of an interval of time of measurement (a moving window) in traffic conditions of the UAV or other vehicle with the demanded speed.

Parameters of various made counters of radiation registration are presented in [6]. In the column "Sensitivity" it is specified, for example: 60-75 imp/μR for the Gamma-7 counter; 285–385 imp/μR for the Gamma-8 counter; 31–39 imp/μR for the Gamma-10 counter. We see, each model allows initial dispersion about 25%. In this regard control of software parameters (sensitivity coefficients) in radiation control devices is required: first of all at a fabrication stage, as well as periodically at an operational phase.

Relevance of article is caused by increase of requirements for accuracy and reliability of measurements in extreme service conditions when replacement of faulty sensitive elements (sensors) is difficult or impossible and there is a need to be protected from their malfunctions and sensitivity drifts.

All sensors are subject to drift, but in a period between sessions of recalculation of sensitivity coefficients excessive drift is most probable only at one sensor. Therefore in this article the object to receive the formulas allowing to analyze (to estimate) influence of drift of one sensor on expected value and dispersion of a median is set.

The solution of that objective depends on a type of probability density function (PDF) of distribution of the values received from sensor. Within this article the uniform (rectangular) distribution is chosen as it is the simply and allows receiving convenient analytical formulas.

INITIAL CONDITIONS

Let $p(a,x)$ — PDF of the measured size x in the field of values which width is characterized by parameter a.

We will write down also cumulative distribution function:

$$P(a,X)=\int_{-\infty}^{X} p(a,x)\,\mathrm{d}x.$$

Let L — drift size towards understating. It means that PDF takes form $p(a,x+L)$.

We will assume that the initial odd number N of identical sensors is used; n is integer half of N: $N=2n+1$.

For example, DRMU [3, 4, 5] contains 24 sensitive elements. In [5] it is reported that in case of existence of a dot source of radiation 17 sensors completely accept its radiation, and 5 sensors, remaining in the cylinder shadow, accept only a background. It is clear, that at the expense of the analysis of such data it is possible to calculate a direction angle on a source. But the main requirement is reliable calculation of a dose rate. Therefore pertinently to ignore 7 smallest values, irrespective of the reason of their replacement to the top of the list of the sorted values, and to consider the remained 17 values for dose rate calculation.

We will consider that the size of a middle interval is equal to 1. As the total measured value we will take the value which has appeared in the middle of the sorted list.

INITIAL CONDITION OF SENSORS (WITHOUT DRIFT)

Any of N sensors can be as middle (with value X) in the sorted list, and in each of two groups of n sensors with values $x<X$ or $x>X$ all variants of sequence are equivalent. We have "shifts with repetitions" [7, p. 48] which number is defined by multinomial coefficient:

$$W(n)=\frac{(2n+1)!}{n!^2}$$

Probability of obtaining value of a median in the range from X to X+dX:

$$Q(a,X,n)\mathrm{d}X=W(n)P(a,X)^n\left(1-P(a,X)\right)^n\times \times p(a,X)\mathrm{d}X. \quad (1)$$

In [8, p. 96] $W(n)$ is presented through binomial coefficient, and in [9, pp. 17–18] — by means of beta function.

Further, to take integrals, we will use property of probability of a totality of events:

$$\int_{-\infty}^{\infty} Q(a,X,n)\mathrm{d}X=1. \quad (2)$$

The expected value of a median (the moment of the first order) has an appearance:

$$\mathrm{M}(1,a,n)=\int_{-\infty}^{\infty} Q(a,X,n)X\,\mathrm{d}X. \quad (3)$$

The moment of the second order of a median (for calculation of a mean square deviation):

$$\mathrm{M}(2,a,n)=\int_{-\infty}^{\infty} Q(a,X,n)X^2\,\mathrm{d}X. \quad (4)$$

We will consider variant of uniform distribution on the set interval. Without restriction of a community for convenience of calculations it is possible to consider that average value is equal to zero. Designate $[-a,a]$ — interval of casual dispersion of the values received from each sensor. PDF has an appearance:

$$p(a,x)=1/(2a) \quad \text{при} \quad -a\le x\le a;$$
$$p(a,x)=0 \quad \text{при} \quad x<-a \quad \text{или} \quad x>a. \quad (5)$$

Integrals are proportional to integration intervals:

$$P(a,X)=(a+X)/(2a); \quad (6)$$

$$1-P(a,X)=(a-X)/(2a). \quad (7)$$

Substituting (6) and (7) in (1), then in (2), taking into account symmetry of integrand expression we receive:

$$2W(n)\int_0^a\left(a^2-x^2\right)^n\mathrm{d}x\Big/(2a)^{2n+1}=1. \quad (8)$$

Further this integral will be required as well in a look:

$$\int_0^a\left(a^2-x^2\right)^n\mathrm{d}x=(2a)^{2n+1}\Big/\left(2W(n)\right). \quad (9)$$

We consider that all sensors are identical. Therefore for calculation of expected value integration of odd function happens evidently:

$$\mathrm{M}(1,a,n)=0.$$

DISPERSION OF THE MEDIAN AT IDENTICAL SENSORS

Symmetry of integrand expression allows to be limited to consideration of positive area:

$$\mathrm{M}(2,a,n)=2W(n)\int_0^a x^2\left(a^2-x^2\right)^n \mathrm{d}x \Big/ (2a)^{2n+1}. \quad (10)$$

Displacing an index n, we will write down (9) in a look:

$$\int_0^a \left(a^2-x^2\right)^{n+1} \mathrm{d}x=(2a)^{2n+3} \Big/ \left(2W(n+1)\right). \quad (11)$$

This integral can be written down also as two integrals:

$$\int_0^a \left(a^2-x^2\right)^{n+1} \mathrm{d}x=$$
$$=a^2\int_0^a \left(a^2-x^2\right)^n \mathrm{d}x-\int_0^a \left(a^2-x^2\right)^n x^2\mathrm{d}x. \quad (12)$$

Equating the right parts of equalities (11) and (12), we receive:

$$\int_0^a \left(a^2-x^2\right)^n x^2\mathrm{d}x=$$
$$=a^2\int_0^a \left(a^2-x^2\right)^n \mathrm{d}x-(2a)^{2n+3} \Big/ \left(2W(n+1)\right). \quad (13)$$

Substituting (9) in (13), we receive:

$$\int_0^a \left(a^2-x^2\right)^n x^2\mathrm{d}x=$$
$$=a^2(2a)^{2n+1} \Big/ \left(2(2n+3)W(n)\right). \quad (14)$$

Substituting (14) in (10), we receive:

$$\mathrm{M}(2,a,n)=\frac{a^2}{2n+3}. \quad (15)$$

We will notice that in [8, p. 101] the formula (15) is presented without proof and without reference to the source. Therefore executed here the receiving of formula (15) is an educational example of application of a formula (9). Further also other formulas will be similarly received.

For comparison we will calculate dispersion of an arithmetic average:

$$\mathrm{m}(2,a,n)=\int_{-a}^{a} x^2\mathrm{d}x \Big/ \left(2a(2n+1)\right)=\frac{a^2}{3(2n+1)}. \quad (16)$$

Comparing (15) and (16), we see that the nature obliges us to pay some cost for our desire to be protected from excessive changes of the resultant value received from a set of sensors. That cost is the excess number of sensors (but no more than three times), or the increased dispersion of values. It while for receiving a formula (15) a condition has been set: the statistical dispersion of the measured values occurs still equally for all sensors, without shifts.

ARITHMETIC AVERAGE IN THE CONDITIONS OF DRIFT

For uniform distribution the displaced PDF has an appearance:

$$p(a,x+L)=1/(2a) \quad \text{при} \quad (-a-L)\le x\le (a-L);$$
$$p(a,x+L)=0 \quad \text{при} \quad \begin{cases} x<(-a-L), \\ x>(a-L). \end{cases} \quad (17)$$

In these conditions, considering the known properties of expected value [10, p. 100], for comparison we will write down expected value of an arithmetic average:

$$\mathrm{m}(1,a,n,L)=-\frac{L}{2n+1}. \quad (18)$$

Considering the known properties of dispersion [10, pp. 103–105], it is easy to be convinced that dispersion of an arithmetic average doesn't depend on L, remains to a constant in the form of (16).

BASIC FORMULAS OF THE MEDIAN IN THE CONDITIONS OF DRIFT

We will designate K — an order of the calculated moment: 1 — for expected value; 2 — for dispersion.

Unlike formulas (1)–(4), now the integral consists from three items, corresponding to three variants of obtaining value from the drifting sensor in comparison with a median:

$$\mathrm{M}(K,a,n,L)=R_1(K,a,n,L)+R_2(K,a,n,L)+$$
$$+R_3(K,a,n,L). \quad (19)$$

1) *Value from the drifting sensor was a median:*

$$R_1(K,a,n,L)=\frac{(2n)!}{n!^2}\times$$
$$\times\int_{-\infty}^{\infty}P(a,X)^n\left(1-P(a,X)\right)^n p(a,X+L)X^K\mathrm{d}X. \quad (20)$$

2) *Value from the drifting sensor has appeared more median:*

$$R_2(K,a,n,L)=\frac{(2n)!}{n!(n-1)!}\times$$
$$\times\int_{-\infty}^{\infty}\Big[P(a,X)^n\left(1-P(a,X)\right)^{n-1}\left(1-P(a,X+L)\right)\times$$
$$\times p(a,X)X^K\Big]\mathrm{d}X. \quad (21)$$

3) *Value from the drifting sensor has appeared less median:*

$$R_3(K,a,n,L)=\frac{(2n)!}{n!(n-1)!}\int_{-\infty}^{\infty}\Big[P(a,X)^{n-1}\times$$
$$\times\left(1-P(a,X)\right)^n P(a,X+L)p(a,X)X^K\Big]\mathrm{d}X. \quad (22)$$

Formulas (19)–(22) are fair for any form of PDF.

FORMULAS OF THE MOMENTS

We will designate:

$$V(n)=\frac{(2n)!}{n!(n-1)!(2a)^{2n+1}}. \quad (23)$$

Taking into account the shift specified in (17) formulas (20)–(22) take a form:

$$R_1(K,a,n,L)=\frac{V(n)}{n}\int_{-a}^{a-L}\left(a^2-X^2\right)^n X^K\mathrm{d}X, \quad (24)$$

$$R_2(K,a,n,L)=$$
$$=V(n)\int_{-a}^{a-L}\left(a^2-X^2\right)^{n-1}(a+X)(a-L-X)X^K\mathrm{d}X, \quad (25)$$

$$R_3(K,a,n,L)=$$
$$=V(n)\int_{-a}^{a-L}\left(a^2-X^2\right)^{n-1}(a-X)(a+L+X)X^K\mathrm{d}X+$$
$$+2aV(n)\int_{a-L}^{a}\left(a^2-X^2\right)^{n-1}(a-X)X^K\mathrm{d}X. \quad (26)$$

Substituting (24)–(26) in a formula (19), we receive:

$$\mathrm{M}(K,a,n,L)=V(n)\int_{-a}^{a-L}\left(a^2-X^2\right)^{n-1}\times$$
$$\times\left(\frac{a^2-X^2}{n}+2\left(a^2-X^2-LX\right)\right)X^K\mathrm{d}X+$$
$$+2aV(n)\int_{a-L}^{a}\left(a^2-X^2\right)^{n-1}(a-X)X^K\mathrm{d}X. \quad (27)$$

We will consider also other view of this expression:

$$\mathrm{M}(K,a,n,L)=$$
$$=V(n)\int_{-a}^{a-L}\left(a^2-X^2\right)^n\left(\frac{1}{n}+2\right)X^K\mathrm{d}X-$$
$$-2LV(n)\int_{-a}^{a-L}\left(a^2-X^2\right)^{n-1}X^{K+1}\mathrm{d}X+$$
$$+2aV(n)\int_{a-L}^{a}\left(a^2-X^2\right)^{n-1}(a-X)X^K\mathrm{d}X. \quad (28)$$

For an expected value, i.e. at K=1, the formula (28) has an appearance:

$$\mathrm{M}(1,a,n,L)=V(n)\int_{-a}^{a-L}\left(a^2-X^2\right)^n\left(\frac{1}{n}+2\right)X\mathrm{d}X-$$
$$-2LV(n)\int_{-a}^{a-L}\left(a^2-X^2\right)^{n-1}X^2\mathrm{d}X+$$
$$+2a^2V(n)\int_{a-L}^{a}\left(a^2-X^2\right)^{n-1}X\mathrm{d}X-$$
$$-2aV(n)\int_{a-L}^{a}\left(a^2-X^2\right)^{n-1}X^2\mathrm{d}X. \quad (29)$$

As one would expect, at L=0 it turns out:

$$\mathrm{M}(1,a,n,0)=0. \quad (30)$$

We won't paint a formula (29) in details in a general view here, we will be limited to consideration of two special cases at which it is possible to apply a formula (14).

At $L=a$ we receive:

$$\mathrm{M}(1,a,n,a)=\frac{a\left(G(n)-1\right)}{2n+1}. \quad (31)$$

Here the amendment which is difference of a formula (31) from a formula (18) is designated:

$$G(n)=\frac{(2n+1)!}{2^{2n+2}n!(n+1)!}.$$

It is easy to receive a recurrent formula:

$$G(n)=G(n-1)(2n+1)/(2n+2). \qquad (32)$$

As an example we will write down several decreasing values:

$$G(0)=\frac{1}{4};\quad G(1)=\frac{3}{16};\quad G(2)=\frac{5}{32};\quad G(3)=\frac{35}{256}.$$

We see that the expected value of a median is always better, than expected value of an arithmetic average. But at small drift (within half-width of statistical dispersion) the received improvement is insignificant, decreasing in process of growth of number of the used sensors.

At $L=2a$ the formula (29) with application of a formula (14) takes a form:

$$M(1,a,n,2a)=-a/(2n+1). \qquad (33)$$

Comparing (33) and (18), we see that when measurements from the drifting sensor have reached border of range of statistical dispersion, the expected value of a median is twice better than an expected value of an arithmetic mean (arithmetic average). It is obvious that at further drift the median doesn't worsen (unlike an arithmetic mean).

It is also clear that this expected value of a median coincides with expected value of an arithmetic mean for a case $L=a$. Therefore the corresponding two columns are presented in table as one general column.

DERIVATIVES OF THE MOMENTS

We will consider a derivative of expression (28) on L. By rules of calculation of a derivative on limits of integral we receive:

$$\mathrm{M}_L(K,a,n,L)=$$

$$=-V(n)(a-L)^K\left(a^2-(a-L)^2\right)^n\left(\frac{1}{n}+2\right)-$$

$$-2V(n)\int_{-a}^{a-L}\left(a^2-X^2\right)^{n-1}X^{K+1}\mathrm{d}X+$$

$$+2LV(n)(a-L)^{K+1}\left(a^2-(a-L)^2\right)^{n-1}+$$

$$+2aV(n)(a-L)^K\left(a^2-(a-L)^2\right)^{n-1}\left(a-(a-L)\right).$$

At L=0 we receive:

$$\mathrm{M}_L(K,a,n,0)=-2V(n)\int_{-a}^{a}\left(a^2-X^2\right)^{n-1}X^{K+1}\mathrm{d}X.$$

At $L=a$ we receive:

$$\mathrm{M}_L(K,a,n,a)=\mathrm{M}_L(K,a,n,0)/2.$$

At $L=2a$ we receive:

$$\mathrm{M}_L(K,a,n,2a)=0. \qquad (34)$$

For an expected value of a median (at K=1), applying a formula (14), we receive:

$$\mathrm{M}_L(1,a,n,0)=\frac{-1}{2n+1}, \qquad (35)$$

$$\mathrm{M}_L(1,a,n,a)=\frac{-1}{2(2n+1)}. \qquad (36)$$

In addition to direct calculations of a function graph on a formula (29), considering simple formulas (30)–(36) for calculations of values of function and its derivatives in three points, we have an opportunity to better understand character of median.

DISPERSION OF THE MEDIAN

The full set of events gives us the chance on the basis of a formula (27), setting an index $n+1$, to write down the following equation at $K=0$: :

$$\mathrm{M}(0,a,n+1,L)=2aV(n+1)\times$$

$$\times\int_{a-L}^{a}\left(a^2-X^2\right)\left(a^2-X^2\right)^{n-1}(a-X)\mathrm{d}X+$$

$$+V(n+1)\int_{-a}^{a-L}\left(a^2-X^2\right)\left(a^2-X^2\right)^{n-1}\times$$

$$\times\left(\frac{a^2-X^2}{n+1}+2\left(a^2-X^2-LX\right)\right)\mathrm{d}X=1.$$

In this expression the moments (at $K=0$ and at $K=2$) set by the same formula (27) are looked through:

$$\frac{1}{V(n+1)} = \left(\frac{1}{n+1} - \frac{1}{n}\right) \int_{-a}^{a-L} \left(a^2 - X^2\right)^{n+1} \mathrm{d}X +$$

$$+ \frac{a^2 \mathrm{M}(0,a,n,L) - \mathrm{M}(2,a,n,L)}{V(n)}.$$

Considering that the full set of events gives us $\mathrm{M}(0,a,n,L) = 1$, in the following step of transformation we receive:

$$\frac{V(n)}{V(n+1)} + \frac{V(n)}{n(n+1)} \int_{-a}^{a-L} \left(a^2 - X^2\right)^{n+1} \mathrm{d}X =$$

$$= a^2 - \mathrm{M}(2,a,n,L). \quad (37)$$

We will rewrite a formula (23) to the displaced index:

$$V(n+1) = \frac{(2n+2)!}{n!(n+1)!(2a)^{2n+3}}.$$

Available in (37) relation takes a form:

$$\frac{V(n)}{V(n+1)} = \frac{2na^2}{2n+1}.$$

Finally from (37) we receive:

$$\mathrm{M}(2,a,n,L) =$$

$$= \frac{a^2}{2n+1} - \frac{V(n)}{n(n+1)} \int_{-a}^{a-L} \left(a^2 - X^2\right)^{n+1} \mathrm{d}X. \quad (38)$$

For a case L=0 considering symmetry of integrand expression, rewriting a formula (9) to the displaced index and substituting in (38), it is easy to be convinced that, as one would expect, the formula (15) turns out.

For L=a case i.e. when PDF of the drifting sensor was displaced on a half of the width, from a formula (38) it is similarly received:

$$\mathrm{M}(2,a,n,a) = \frac{a^2(2n+2)}{(2n+1)(2n+3)}. \quad (39)$$

Substituting (31) and (39) in a dispersion calculation formula [10, p. 103], we receive:

$$\mathrm{M}(2,a,n,a) - \mathrm{M}(1,a,n,a)^2 =$$

$$= a^2 \frac{2n + \frac{2}{2n+3} - \left(G(n) - 1\right)^2}{(2n+1)^2}. \quad (40)$$

For L=2a case i.e. when PDF of the drifting sensor was displaced out of limits of range of dispersion of values of all other sensors of group, from a formula (38) we receive:

$$\mathrm{M}(2,a,n,2a) = a^2/(2n+1). \quad (41)$$

Substituting (33) and (41) in a dispersion calculation formula [10, p. 103], we receive:

$$\mathrm{M}(2,a,n,2a) - \mathrm{M}(1,a,n,2a)^2 = \frac{2na^2}{(2n+1)^2}. \quad (42)$$

The difference between (42) and (15) is

$$a^2 \frac{2n-1}{(2n+1)^2(2n+3)}$$

and allows us to notice that in process of drift of one of sensors dispersion of a median increases.

RESULTS OF CALCULATIONS ON THE RECEIVED FORMULAS

The calculated data of the expected value (EV) and mean square deviation (MSD) of a median and the arithmetic mean (AM) for the rated range of random values of the uniform law of distribution are presented in the table (we consider that $a = 1$).

Parameters of a median and arithmetic mean for the uniform law of distribution

$N = 2n+1$	EV			MSD			
	median	median, AM	AM	median			AM
	a	2*a* (median), *a* (AM)	2*a*	0	*a*	2*a*	
3	0.27083	0.33333	0.66667	0.44721	0.43968	0.47140	0.33333
5	0.16875	0.,20000	0.40000	0.37796	0.37809	0.40000	0.25820
7	0.12333	0.14286	0.28571	0.33333	0.33433	0.34993	0.21822
9	0.09744	0.11111	0.22222	0.30151	0.30252	0.31427	0.19245
11	0.08066	0.09091	0.18182	0.27735	0.27823	0.28748	0.17408
13	0.06887	0.07692	0.15385	0.25820	0.25894	0.26647	0.16013
15	0.06012	0.06667	0.13333	0.24254	0.24317	0.24944	0.14907
17	0.05337	0.05882	0.11765	0.22942	0.22996	0.23529	0.14003
19	0.04799	0.05263	0.10526	0.21822	0.21868	0.22330	0.13245
21	0.04361	0.04762	0.09524	0.20851	0.20892	0.21296	0.12599

CONCLUSION

1. The relation of an expected value of a median EV_{med} to an expected value of an arithmetic mean EV_{AM} reaches the level 1/2 when PDF of the drifting sensor goes beyond PDF of other sensors.

2. At the same time the sums $EV_{med} + MSD_{med}$ and $EV_{AM} + MSD_{AM}$ are approximately identical, differing within 15%. It means that in drift conditions use of a median with its property of broader dispersion not only is justified, but also doesn't create feeling of decline in quality of the obtained information for the user.

3. On the basis of the received formulas and results of calculations an opportunity to form expert opinion on the necessary number of sensors is given.

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